NAG Toolbox for MATLAB

f07uu

1 Purpose

f07uu estimates the condition number of a complex triangular matrix, using packed storage.

2 Syntax

```
[rcond, info] = f07uu(norm_p, uplo, diag, n, ap)
```

3 Description

f07uu estimates the condition number of a complex triangular matrix A, in either the 1-norm or the ∞-norm, using packed storage:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \qquad \text{or} \qquad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{T}}\right)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham 1988) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p – string**

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo – string**

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n triangular matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether **diag** = 'N' or 'U'.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

work, rwork

5.4 Output Parameters

1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **uplo**, 3: **diag**, 4: **n**, 5: **ap**, 6: **rcond**, 7: **work**, 8: **rwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07uu involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $4n^2$ real floating-point operations but takes considerably longer than a call to f07us with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is f07ug.

9 Example

```
norm_p = '1';
uplo = 'L';
diag = 'N';
n = int32(4);
ap = [complex(4.78, +4.56);
     complex(2, -0.3);
     complex(2.89, -1.34);
     complex(-1.89, +1.15);
     complex(-4.11, +1.25);
     complex(2.36, -4.25);
     complex(0.04, -3.69);
     complex(4.15, +0.8);
     complex(-0.02, +0.46);
     complex(0.33, -0.26)];
[rcond, info] = f07uu(norm_p, uplo, diag, n, ap)
```

```
rcond =
    0.0268
info =
           0
```